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The Empirics of Okun's Law Using Augmented ARDL: The Case of Pakistan

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Abstract

From time series data covering 1972 to 2024, the objective of the study is to investigate the Okun's coefficient for Pakistan. The coefficient of Okun's law is determined using the Augmented ARDL econometric technique to estimate the long-run elasticities and short-run dynamics while sidestepping degenerate cases. Four variations of Okun's law—the gap, difference, dynamic version, and augmented production function versions—are applied to estimate the Okun's coefficient. In order to measure output and unemployment gap, Hodrick-Prescott (HP) filter is also used. The empirical results demonstrated that, in the case of Pakistan, the law doesn't hold, as evidenced by the comparatively low coefficients obtained when applying all four versions.

Keywords: Okun's law, Gap version, Difference version, Dynamic version, Unemployment, Potential Output, Augmented ARDL, Hodrick-Prescott (HP) filter

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1. Introduction

Okun's law describes the empirical relationship between changes in economic output and changes in the unemployment rate. In general, unemployment tends to decline when output increases and to rise when output falls. The relationship is named after Arthur Okun, who examined it in 1962 (Okun, 1962). However, the size and stability of the estimated relationship vary across countries and periods. Therefore, the coefficient should be interpreted in relation to the particular model specification and measurement units rather than compared with a single universal value.

A stable relationship between output and unemployment can inform macroeconomic policy decisions. Fiscal and monetary measures that support aggregate demand may encourage firms to expand production and employment. However, the employment response also depends on labor market institutions, workforce skills, and hiring conditions. In addition, the slow recovery of employment following major economic disruptions, including the COVID-19 pandemic, has renewed interest in the relationship between output growth and unemployment (Hussain & Malik, 2013).

Many empirical studies identify an inverse relationship between output and unemployment. However, the magnitude and stability of the estimated Okun coefficient remain contested. Estimates differ across countries, sample periods, and methods (Ball et al., 2019; Ball et al., 2017). Therefore, evidence from Pakistan must be interpreted in the context of its economic structure and labour market conditions.

Studies of Okun's law in Pakistan have produced mixed findings. Differences in economic structure, sample periods, and empirical methods may help explain this variation. For example, agriculture and informal employment play important roles in Pakistan, while external shocks, including commodity price movements and weather-related events, may influence both production and employment (Hussain & Malik, 2013). Some studies report the expected inverse relationship between output and unemployment. Khan and Qayyum (2012), for example, identify a negative association between the unemployment gap and the output gap, while Hussain and Malik (2013) also report an inverse relationship. However, Khan and Qayyum (2007) find a weak and statistically insignificant relationship for their study period. These contrasting results indicate the need for careful comparison of empirical specifications.

Batavia and Abdussalam (2012) examined the relationship between unemployment and gross domestic product in Pakistan and questioned the applicability of Okun's law. Similarly, Akram et al. (2014) estimated the gap, difference, and dynamic forms using ordinary least squares and reported relatively small coefficients. Ahmad et al. (2015) also questioned the law's relevance to Pakistan (cite here). In contrast, Abbas (2014) reported an inverse and statistically significant long-run relationship but no significant short-run association (cite here). Therefore, the existing literature does not provide a single consistent conclusion.

Khan et al. (2015) reported a long-run relationship between unemployment and GDP but found no evidence of a short-run relationship (cite here). Hussain and Malik (2016), however, identified a short-run association without clear evidence of a long-run relationship (Hussain & Malik, 2013). Similarly, Ali and Ahmed (2017) reported a weak relationship for 1981–2014 and suggested that labour market informality could be relevant. Khan et al. (2020) found evidence of a long-run but not a short-run relationship. In addition, Shah et al. (2022) reported an inverse association between unemployment and economic growth in Pakistan. These differences provide a basis for evaluating alternative specifications within a common dataset.

This study examines the relationship between output and unemployment in Pakistan using four specifications of Okun's law: the gap, difference, dynamic, and production-function forms. It applies the augmented autoregressive distributed lag (ARDL) approach to examine long-run relationships and short-run dynamics while allowing for tests of degenerate cases. In addition, the study draws on an extended annual sample, reported as 1972–2024. By considering the four specifications within a common empirical framework, the analysis provides a basis for comparing how alternative definitions and dynamic assumptions affect the estimated output–unemployment relationship. Section 2 describes the data and methods, Section 3 develops the specifications, Section 4 discusses the estimates, and Section 5 presents the conclusions.

2. Research Method and Data Specification

2.1. Introduction

The study draws on annual time-series data for Pakistan, including real gross domestic product (GDP), the unemployment rate, gross fixed capital formation as a proxy for physical capital, the labour force, and a weighted education-enrolment index as a proxy for human capital. The sample covers 1972–2024, and the data come from the World Development Indicators (WDI) and the State Bank of Pakistan (SBP). The variables are reported in natural logarithms. However, the treatment of zero or negative gap observations and the specific series used for each variable remain to be clarified (Hussain & Malik, 2013).

The gap specification requires estimates of potential output and the natural rate of unemployment, in addition to observed GDP and unemployment. Available methods for identifying trends and cyclical components include the Kalman, Baxter–King, and Hodrick–Prescott (HP) filters. This study uses the HP filter, consistent with earlier empirical applications (Hodrick & Prescott, 1997; Ting & Ling, 2011). The choice of filter may influence the resulting gaps, particularly near the beginning and end of the sample.

$$min_{\varphi} \sum_{t=1}^{T} (\gamma_T - \varphi_t)^2 + \psi \sum_{t=2}^{T} [(\varphi_{t+1} - \varphi_t) - (\tau\varphi_t - \varphi_{t-1})]^2$$

The first component of the HP-filter objective measures deviations between the observed series and its estimated trend, whereas the second penalises changes in the trend's growth rate. For these annual observations, the smoothing parameter is set to 100. However, alternative parameter values may produce different estimates of potential output and the unemployment trend (Hussain & Malik, 2013).

2.2. Potential Endogeneity Problems

Endogeneity may arise from omitted variables, measurement error, or feedback between output and unemployment. Consequently, estimated coefficients may be biased if explanatory variables are correlated with the disturbance term. Barreto and Howland (1993) discuss alternative formulations of the output–unemployment relationship. However, lagging regressors does not necessarily eliminate simultaneity or reverse causality; this remains a limitation when interpreting the estimated associations.

Clemens et al. (2012) highlight the importance of timing assumptions in empirical estimation, while Huang et al. (2020) examine endogeneity in Okun's law. In the present study, we use lagged regressors instead of an instrumental-variable specification. This approach captures delayed adjustment and may reduce some contemporaneous feedback. Nevertheless, we interpret the estimates as associations rather than causal effects because the lag structure does not, by itself, establish causal identification.

3. Data

We obtain unemployment, GDP, and the GDP deflator from the WDI. We estimate potential output and the unemployment trend using the HP filter, then calculate the corresponding gaps. The output gap is the difference between actual and potential output, while the unemployment gap is the difference between observed unemployment and its estimated trend. These measures are retained in the model specifications, with their interpretation depending on whether the variables enter in levels, percentages, or transformed form.

3.1 Theoretical Framework and Specifications of Okun's Law

Okun's law can be represented through several specifications that differ in how output and unemployment are measured and how delayed adjustments are modelled. Accordingly, each specification produces a coefficient whose interpretation depends on its functional form and units.

The study considers the following four specifications:

- The Gap Form
- The Difference Form

- The Dynamic Form
- The Production-Function Form

3.2. The Gap Form

In the gap specification, the difference between actual and potential output is related to the difference between observed unemployment and its estimated natural rate. Okun (1962) provided an early formulation of this relationship, which empirical research has since extended (Perman et al., 2015). The specification used here is presented below:

$Ou_t - Ou_t^* = \alpha + \beta_1 (UE_t - UE_t^*) + \epsilon_t$ ……. 1

In Equation (1), Out denotes actual output; Out* denotes potential output; UEt is the observed unemployment rate; UEt* is the estimated natural rate; εt is the disturbance term; and β1 is the estimated Okun coefficient.

A negative coefficient indicates that a larger unemployment gap is associated with a smaller output gap. However, both the magnitude and statistical significance of the estimate depend on the specification and the units of measurement. In addition, productivity changes, measurement error, and omitted factors may affect the estimated relationship (Hussain & Malik, 2013).

3.3. The Difference Form

The difference specification relates changes in output to changes in unemployment between successive periods. It is written as follows:

Out – Out-1 = αo + α1 (UEt – UEt-1) + et ……. (2)

In Equation (2), Out and Out−1 represent output in periods t and t−1, respectively, while UEt and UEt−1 represent the unemployment rates in the corresponding periods.

Here, et is the disturbance term, α1 is the estimated Okun coefficient, and α0 is the intercept. Unlike the gap specification, the difference form does not require a direct estimate of potential output. An inverse relationship is generally expected between changes in output and unemployment. However, changes in GDP levels should be distinguished from its growth rate because these measures have different economic interpretations. The time-series properties of the transformed variables also matter for the estimation procedure.

3.4. The Dynamic Form

The dynamic specification allows the output–unemployment relationship to reflect both current and lagged adjustments. Firms may initially respond to output shocks by changing hours worked or labour utilisation rather than immediately adjusting employment. Therefore, contemporaneous specifications may omit relevant delayed responses (Knotek, 2007; Owyang & Sekhposyan, 2012; Ball et al., 2017). The dynamic model is specified as follows:

Out – Out* = β0 + β1 (UEt – UEt*) + β2 (Out-1 – Ou*t-1) + β3 (Out-2 – Ou*t-2) + δ1(UEt-1 – UE*t-1) + δ2 (UEt-2 – UE^*_{t-2}) + ϵ_t ……..…… 3

Equation (3) incorporates current and lagged output and unemployment gaps to account for delayed responses. The intercept, slope parameters, and disturbance term retain their usual meanings. The dynamic structure therefore extends the contemporaneous association to include adjustment across successive periods. The dynamic specification extends the static gap formulation by including lagged values. In this way, it links current output gaps to current and previous unemployment gaps and to the output gap's history.

Equation (3) provides a more flexible representation of the gap relationship. Including lags may capture delayed adjustment and reduce residual serial correlation, although these properties are assessed empirically rather than assumed. In addition, the resulting coefficients are interpreted in the context of the full dynamic specification. The ARDL framework examines possible long-run relationships alongside short-run adjustments. It permits different lag lengths across variables and can accommodate a mixture of I(0) and I(1) regressors, provided that none is I(2) and the relevant assumptions are satisfied

(Pesaran et al., 2001). The augmented bounds-testing approach also considers whether the additional tests used to assess degenerate cases support a level relationship.

3.5. The Production Function Form

Output is produced through a combination of labour, capital and other productive resources. Therefore, unemployment may capture only part of the variation in resource utilisation. Following the production-function approach discussed by Perman et al. (2015), the study includes physical capital, labour force and human capital alongside the unemployment gap. The proposed specification is:

$$\ln (Out - Out^*) = \beta_0 + \beta_1 K_t + \beta_2 L_t + \beta_3 HC_t + \beta_4 (UE_t - UE_t^*) \text{ ---------- (4)}$$

In Equation (4), Out denotes actual output, Out* potential output, Kt physical capital, Lt the labour force, HCt the human-capital proxy, and UEt−UEt* the unemployment gap. The β parameters represent the estimated coefficients. The logarithmic transformation of the output gap is problematic when the gap is zero or negative; therefore, interpreting this specification depends on how the gap variable was constructed.

4. Results and Discussion

4.1. Stationarity Test

We examined the time-series properties of the variables using the augmented Dickey–Fuller (ADF) and Phillips–Perron (PP) tests. Both tests have a unit-root null hypothesis. As reported in Table 1, the unemployment gap (UEG), capital (K) and labour force (L) are classified as stationary in levels, whereas the GDP-related measure and human capital (HC) are classified as I(1). However, the significance symbols reported for human capital are inconsistent with its stated integration order, so this classification remains uncertain.

Table 1: Stationarity Test Results

	At Level Intrcpt	Trnd & Intrcpt	At 1st Diff Intrcpt	Trnd & Intrcpt	Decision
Augmented Dick-Fuller (ADF) Test					
GPGC	-1.92	-1.89	-5.39**	-7.16**	I (1)
UEG	-3.91**	-4.28**	…	…	I (0)
K	-3.44**	-3.46*	…	…	I (0)
L	-14.3**	-16.5**	…	…	I (0)
HC	-0.93**	-1.74	-6.19	-6.16**	I (1)
Philips Perron (PP) Test -1.96					
GPGC		-1.83	-5.13	-7.14**	I (1)
UEG	-3.91**	-4.33**	…	…	I (0)
K	-3.97**	-3.72*	…	…	I (0)
L	-9.19**	-13.7**	…	…	I (0)
HC	-0.92	-1.74	-6.17	-6.14**	I (1)

Source: Calculated by the authors. ** and * mean significance at 1% and 5%, respectively.

4.2. Gap Form

Table 2: ARDL Bounds Test Results

F Bounds Test		**H_0: No level relationship**		
		Significance.	**I(0)**	**I(1)**
F Statistic	8.929473	10%	3.02	3.51
		5%	3.62	4.16
		1%	4.94	5.58

Source: Calculated by the authors.

The F-statistic of 8.93 exceeds the reported 1% upper-bound critical value of 5.58. Therefore, the bounds test rejects the null hypothesis of no level relationship under the stated assumptions. The additional augmented ARDL tests are considered separately when assessing whether this relationship represents a non-degenerate cointegrating association.

Table 3: ARDL Long-Run Coefficients

ARDL Long Run				
Dependent Varble: D(LOGGDPGAP)				
Selected Model: ARDL (5, 2)				
Variable	**Coefficient**	**Std. Error**	**t-Statistic**	**Prob.**
UEGGAP	-0.001501	0.000926	-1.621017	0.1137
C	-7.97E-05	0.000291	-0.274224	0.7855

Source: Calculated by the authors.

Table 4: Diagnostic Results for the Gap Form

Nature of the problem	**Test-statistics**	**p-value**
Specification (Ramsey RESET Test)	1.466325	0.245
Normality (Jarque-Bera)	0.393047	0.821
Heteroskedasticity	9.513	0.950
Autocorrelation	0.950600	0.396

Source: Calculated by the authors.

Charts of the dynamic stability of all four versions of the law are given in the appendices as Appendix.

Table 5: ARDL Short Run Coefficients

Variable		**Standard Error**	**t-Statistic**	**Prob.**
Δ (LGDPGAP(-1))	1.075836	.300379	3.581600	0.0010
Δ (LGDPGAP(-2))	.733425	.258041	2.842286	0.0073
Δ (LGDPGAP(-3))	.464172	.219022	2.119298	0.0410
Δ (LGDPGAP(-4))	.354749	.156478	2.267078	0.0295
Δ (UEGGAP)	-.000761	.000677	-1.124386	0.2683
Δ (UEGGAP(-1))	.001834	.000748	2.452155	0.0192

Variable		Standard Error	t-Statistic	Prob.
ECM(-1)*	-.912619	.359678	-5.317581	0.0000
Goodness of Fit R^2	0.559249	Durbin-Watson	stat	2.061707
Adjusted R^2	0.489657	S.D. dependent var		0.005826

Source: Calculated by the authors.

The gap specification yields a reported long-run unemployment-gap coefficient of −0.001501 (p = 0.1137). Although its sign is negative, the estimate is not statistically significant at the 5% level. Therefore, this specification does not provide strong evidence of a long-run gap relationship in the reported sample. The coefficient should not be compared directly with a fixed benchmark of two or three because its numerical magnitude depends on the dependent variable and on the measurement of the output and unemployment gaps. In addition, labour market characteristics may influence how changes in output translate into unemployment (Hussain & Malik, 2013). The negative error-correction coefficient is consistent with adjustment towards equilibrium. However, the reported coefficient, standard error, and t-statistic in Table 5 are inconsistent, so the implied adjustment rate cannot be interpreted reliably from the presented figures.

4.3. Difference Form

Table 6: ARDL Bounds Test Results

F Bounds Test		H_0: No level relationship		
		Significant.	I(0)	I(1)
F-stat	139.6600	10%	3.02	3.51
		5%	3.62	4.16
		2.5%	4.18	4.79
		1%	4.94	5.58

Source: Calculated by the authors.

Table 7: ARDL Long Run Coefficients

ARDL Long Run Form Depndt Variable: D(LLGDP) Selected Model: ARDL(1, 1) Variable	Standard Error		t-Stat	P-value.
LUEG	0.013994	0.116011	0.120629	0.9045
C	8.466627	1.083042	7.817448	0.0000

Source: Calculated by the authors.

Table 8: Diagnostic Checking for the Difference Form

Nature of the problem	Test-stat	p-value
Specification (Ramsey RESET Test)	0.134	0.715
Normality (Jarque-Bera)	0.856	0.651
Heteroskedasticity	0.348	0.950
Autocorrelation	0.525	0.594

Source: Calculated by the authors.

Table 9: ARDL Short Run Coefficients

ARDL Error Correction **Dependent Variable: D(LLOGGDP)** **Selected Model: ARDL(1, 1)**				
Variable		**Standard Error**	**t-Stat**	**P-Value.**
Δ (LUEG)	-.001695	.000838	-2.023134	0.0493
ECM(-1)* Goodness of Fit	-.010949	.000523	-20.93962	0.0000
R^2	0.233010	S.D. dependent var		0.007821
Adjusted R^2	0.215966	DW		1.734161

Source: Calculated by the authors.

The difference specification reports a long-run unemployment coefficient of 0.013994 (p = 0.9045), which is not statistically significant. However, the short-run coefficient for changes in unemployment is negative (−0.001695) and significant at the 5% level (p = 0.0493). Therefore, this model provides different evidence for the short-run and long-run relationships. The contrast may reflect differences in the transformation and lag structure and is consistent with the possibility of varying adjustment across time horizons. The reported error-correction coefficient is −0.010949 and statistically significant, suggesting a relatively slow adjustment towards equilibrium if the model is correctly specified. Nevertheless, the long-run interpretation remains conditional on the validity of the transformed variables and the augmented bounds-testing results.

4.4. Dynamic Form

Table 10: ARDL Bounds Test Results

F Bounds Test		**H_0: No level relationship**		
		Significance.	**I(0)**	**I(1)**
F-stat	5.04207	10%	2.08	3
	5%		2.39	3.38
	2.5%		2.7	3.73
	1%		3.06	4.15

Source: Calculated by the authors.

Table 11: ARDL Long-run Coefficients

ARDL Long Run Form **Dependent Variable: D(LGDPGAP)** **Selected Model: ARDL(3, 0, 0, 0, 0, 0)**				
Variable	**Coefficient**	**Std. Error**	**t-Statistic**	**Prob.**
UEGGAP	-0.000014	1.83E-05	0.767812	0.4475
LGDPGAP	1.308311	0.023867	54.81758	0.0000
L2GGDPGAP	-0.291823	0.017329	-16.83975	0.0000
LUEGGAP	-6.65E-06	1.34E-05	-0.496245	0.6227
L2UEGGAP	1.64E-05	1.55E-05	1.055477	0.2981
C	5.46E-06	8.42E-06	0.648482	0.5207

Diagnostic Checking for The Dynamic Form

ARDL Long Run Form Dependent Variable: D(LGDPGAP) Selected Model: ARDL(3, 0, 0, 0, 0, 0)				
Variable	**Coefficient**	**Std. Error**	**t-Statistic**	**Prob.**
Nature of the problem			**Test-statistics**	**p-value**
Specification (Ramsey RESET Test)			1.010	0.319
Normality (Jarque-Bera)			0.348	0.840
Heteroskedasticity			6.319	0.610
Autocorrelation			1.292	0.287

Source: Calculated by the authors.

Table 12: ARDL Short-Run Coefficients

ARDL ECM Dependent Variable: D(LGDPGAP) Selected Model: ARDL(3, 0, 0, 0, 0, 0)				
Variable		**Standard Error**	**t-Stat**	**P-value.**
Δ (LGDPGAP(-1))	20.6861	3.270561	6.324966	0.0000
Δ (LGDPGAP(-2))	0.257529	0.138962	1.853229	0.0718
ECM(-1)*	-0.34186	10.82700	6.404529	0.0000
Goodness of Fit R^2	0.546010	S.D. dependent var		0.005794
Adjusted R^2	0.524894	DW		2.045211

Source: Calculated by the authors.

The dynamic specification reports an unemployment-gap coefficient of −0.000014 (p = 0.4475). The estimate is negative but not statistically significant at the 5% level. However, the specification includes additional lagged output and unemployment terms, and the relationship therefore cannot be assessed solely from this contemporaneous coefficient. The results suggest that the estimated association is sensitive to the way dynamic adjustments are modeled. The dynamic model reports an error-correction coefficient of −0.34186, which is consistent in sign with adjustment towards equilibrium. However, the corresponding standard error and t-statistic in Table 12 do not agree with the reported coefficient. Consequently, the published figures do not allow a precise speed of adjustment to be established.

4.5. The Production Function Form

Table 13: ARDL Bounds Test Results

F Bounds Test		**H_0: No level relationship**		
		Significance.	**I(0)**	**I(1)**
F-stat	6.91	10%	2.2	3.09
		5%	2.56	3.49
		1%	3.29	4.37

Source: Calculated by the authors.

Table 14: ARDL Long-Run Coefficients

ARDL Long Run Form **Dependent Variable: D(LGDPGAP)** **Selected Model: ARDL(5, 5, 1, 5, 5)** **Variable**		**Standard Error**	**t-Stat**	**P-Value.**
UEGGAP	-0.004136	0.001695	-2.439747	0.0247
L	0.033687	0.009925	3.394277	0.0030
K	0.003734	0.003254	-1.147682	0.2653
HC	0.013374	0.007933	-1.685718	0.1082
C	-0.153396	0.060070	-2.553602	0.0194

Source: Calculated by the authors.

Table 15: Diagnostic Checking for the Production Function Form

Nature of the problem	**Test-statistics**	**p-value**
Specification (Ramsey RESET Test)	0.006	0.938
Normality (Jarque-Bera)	0.835	0.658
Heteroskedasticity	27.083	0.819
Autocorrelation	0.006	0.927

Source: Calculated by the authors.

Table 16: ARDL Short-Run Coefficients

ECM **Dependent Variable: D(LGDPGAP)** **Selected Model: ARDL (4, 4, 1, 4, 4)** **Variable**		**Standard Error**	**t-Stat**	**P-value**
ΔLDPGAP(-1))	2.097802	0.363479	5.771449	.0000
Δ(LGDPGAP(-2))	1.812606	0.316633	5.724636	.0000
Δ (LGDPGAP(-3))	1.339436	0.253471	5.284373	.0000
Δ (LGDPGAP(-4))	0.694277	0.163623	4.243146	.0004
Δ (UEGGAP)	0.001119	0.000773	1.448755	.1637
Δ (UEGGAP(-1))	0.013449	0.001946	6.912305	.0000
Δ (UEGGAP(-2))	0.009894	0.001524	6.491745	.0000
Δ (UEGGAP(-3))	0.005315	0.000970	5.480553	.0000
Δ (UEGGAP(-4))	0.002891	0.000666	4.339218	.0004
Δ (L)	0.430529	0.089221	4.825411	.0001
Δ (K)	-5.53E-05	0.024159	-0.002289	.9982

ECM Dependent Variable: D(LGDPGAP) Selected Model: ARDL (4, 4, 1, 4, 4)				
Variable		**Standard Error**	**t-Stat**	**P-value**
Δ (K(-1))	-0.056442	0.025050	-2.253174	.0363
Δ (K(-2))	0.060341	0.024866	2.426629	.0254
Δ (K(-3))	0.074969	0.028052	2.672520	.0151
Δ (K(-4))	0.026897	0.020955	1.283570	.2147
Δ (HC)	-0.110180	0.026895	-4.096680	.0006
Δ (HC(-1))	0.099546	0.031746	3.135744	.0054
Δ (HC(-2))	0.110231	0.034370	3.207191	.0046
Δ (HC(-3))	0.000623	0.031574	0.019742	.9845
Δ (HC(-4))	-0.084269	0.034022	-2.476875	.0228
ECM(-1)*	-0.032867	0.418943	-7.239328	.0000
Goodness of Fit R^2	0.847743	S.D. dependent var		.005826
Adjusted R^2	0.720863	DW		2.055740

Source: Calculated by the authors.

In the production-function specification, the unemployment-gap coefficient is negative (−0.004136) and statistically significant ($p = 0.0247$). This finding is consistent with an inverse relationship between output and unemployment, although its economic magnitude depends on the measurement units. The labour-force coefficient is positive and statistically significant, whereas the estimated coefficients for physical and human capital are reported as statistically insignificant. However, the signs of some reported t-statistics are inconsistent with their coefficients. The production-function estimates therefore provide qualified, rather than uniform, evidence concerning Okun's law in Pakistan. The reported error-correction coefficient in the production-function model is −0.032867, suggesting gradual adjustment towards equilibrium. However, its standard error and t-statistic are inconsistent with the coefficient, limiting any quantitative interpretation of the adjustment rate. Earlier studies, including Akram et al. (2014) and Batavia and Abdussalam (2012), provide points of comparison. In addition, characteristics of Pakistan's labour market offer possible explanations for the differences between specifications, although these mechanisms are not directly identified by the present estimates.

Informal employment: Pakistan has a substantial informal sector, which may weaken the correspondence between measured unemployment and changes in output. Informal and low-productivity activities may also be imperfectly captured in official statistics (Mughal & Schneider, 2020; Kiani et al., 2014). Similarly, the Pakistan Labour Force Survey 2020–21 documents the importance of informal employment outside agriculture. Differences in coverage and definitions should therefore be considered when interpreting labour market indicators. Underemployment: Some workers are employed for fewer hours than they would prefer or in roles that do not fully use their skills. Consequently, unemployment rates alone may not capture all forms of labour underutilisation. This may be relevant when interpreting the estimated output–unemployment relationship (Akram et al., 2014).

Skills mismatch: A mismatch between workers' skills and employers' requirements may constrain the employment response to output growth. Therefore, unemployment may persist even when economic activity expands. Estevão and Tsounta (2011) discuss structural unemployment and related labour market factors, although evidence specific to Pakistan remains necessary (Batavia and Abdussalam, 2012). Population growth: Growth in the working-age population can increase labour supply and the

number of individuals seeking employment. If job creation does not keep pace, unemployment or underemployment may persist despite output growth. In addition, the effects of demographic change are likely to depend on labour-force participation and the structure of employment (Akram et al., 2014).

5. Conclusion

This study examines the relationship between economic output and unemployment in Pakistan through the gap, difference, dynamic, and production-function specifications of Okun's law. The data are drawn from the WDI and SBP for the study period reported as 1972–2024. The estimates differ across specifications. The gap and dynamic models report statistically insignificant unemployment-gap coefficients; however, the difference model identifies a statistically significant negative short-run coefficient, and the production-function model reports a significant negative long-run unemployment-gap coefficient. Therefore, the findings suggest that the estimated output–unemployment relationship in Pakistan depends on the specification and time horizon rather than supporting a uniform rejection of Okun's law. These interpretations remain subject to the inconsistencies in the reported econometric tables. Several characteristics of Pakistan's economy may affect the relationship between output changes and recorded unemployment. These include informal employment, agriculture, underemployment, skills mismatch and limitations in labour market measurement. However, the current models do not directly identify the causal contributions of these factors. They are therefore considered possible explanations for variation in the estimated coefficients rather than established empirical mechanisms (Hussain & Malik, 2013).

6. Data Limitations and Policy Recommendations

Reliably measuring unemployment and GDP can be challenging where informal work and incomplete reporting are prevalent. Consequently, measurement error may affect the estimated relationship between output and unemployment. In addition, the HP filter and alternative variable transformations may influence the results. These limitations should be considered when interpreting the estimated coefficients, and future work could test the robustness of the findings using alternative measures and specifications.

The findings have implications for the interpretation of labour market indicators and the design of employment policy. First, more comprehensive statistics on informal employment and underemployment would strengthen assessments of labour market responses to economic growth. Second, investment in skills development and productive employment may help address structural constraints, although the present estimates do not establish the effects of specific interventions. In addition, future research could investigate differences across economic sectors, periods of expansion and contraction, and episodes of major economic disruption. Such analyses could help explain why the estimated relationship varies across the four specifications.

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Appendices

Appendix 1

Augmented ARDL Bound Test for the Gap form

Exogenous F-Bounds Test			**Null Hypothesis: No exo. levels relationship**		
F-statistic = 18.284			t-statistic= -5.168032		
10%	2.87	5.30	10%	-2.57	-2.91
5%	4.11	7.33	5%	-2.86	-3.22
1%	7.41	12.13	1%	-3.43	-3.82

Augmented ARDL Bound Test for the Difference form

Exogenous F-Bounds Test			**Null Hypothesis: No exo. levels relationship**		
F-statistic = 14.020780			t-statistic= -5.784473		
10%	2.87	5.30	10%	-2.57	-2.91
5%	4.11	7.33	5%	-2.86	-3.22
1%	7.41	12.13	2.5%	-3.13	-3.5
			1%	-3.43	-3.82

Augmented ARDL Bound Test for the Dynamic form

Exogenous F-Bounds Test			**Null Hypothesis: No exo. levels relationship**		
F-statistic = 7.292491			t-statistic= -5.073265		
10%	2.00	3.57	10%	-2.57	-3.86
5%	2.45	4.25	5%	-2.86	-4.19
1%	3.46	5.81	1%	-3.43	-4.79

Augmented ARDL Bound Test for the Production form

Exogenous F-Bounds Test			**Null Hypothesis: No exo. levels relationship**		
F-statistic = 6.265647			t-statistic= -6.085517		
10%	2.08	3.76	10%	-2.57	-3.66
5%	2.60	4.55	5%	-2.86	-3.99
1%	3.80	6.33	1%	-3.43	-4.6

All model forms show no degenerate case; therefore, a cointegration relationship exists.

Appendix 2: CUSUM and CUSUM Squares Charts

Chart 1: dynamic stability of the Gap form model parameters

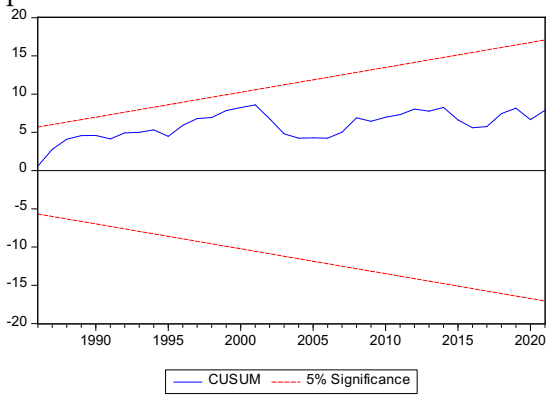


Chart 2: dynamic stability of the Gap form model parameters

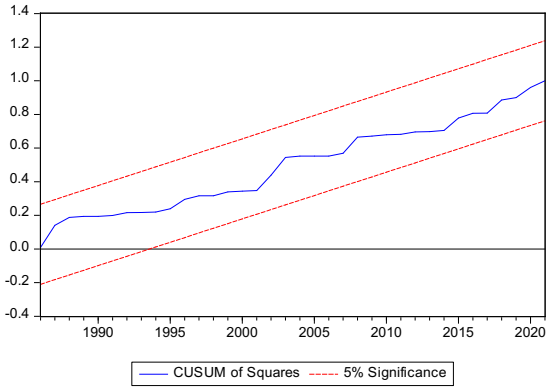


Chart 3: Dynamic stability of the difference form of the model parameters

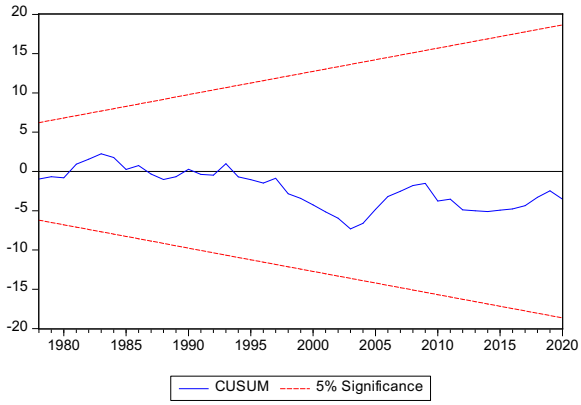


Chart 4: Dynamic stability of the difference form of the model parameters

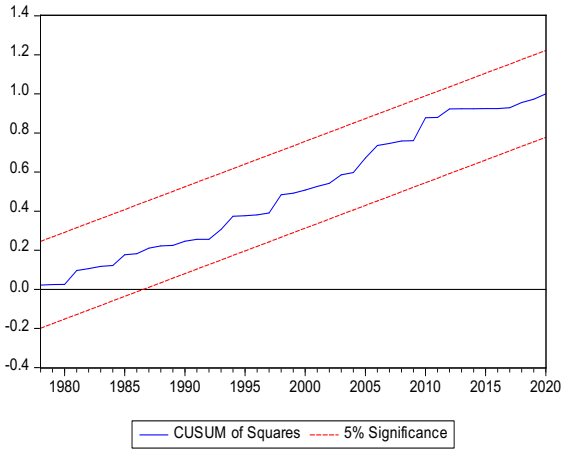


Chart 5: Dynamic stability of the dynamic form of the model parameters

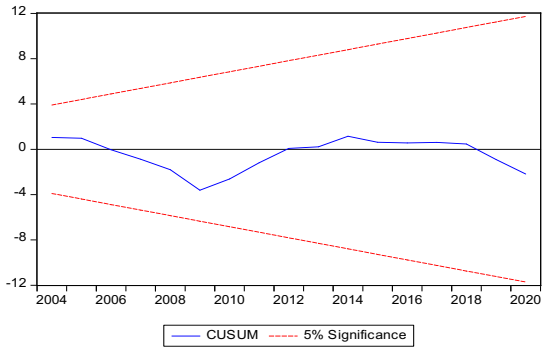


Chart 6: Dynamic stability of the dynamic form of the model parameters

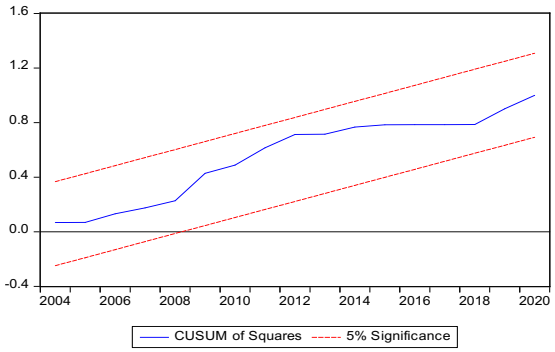


Chart 7: Dynamic stability of the production function form of the model parameters

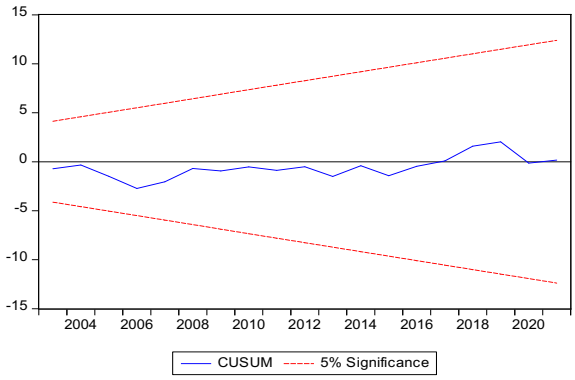


Chart 8: Dynamic stability of the production function form of the model parameters

